

## A New Method for Calculating of Electric Fields Around Any Arbitrary Shape Electrodes

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**Abstract:** In this paper, a new method for calculating of electric fields around any arbitrary shape electrode have been represented. In research and educational activities, we can use also this computer code for calculating electric field intensity and equipotential lines around any arbitrary shape electrode. This computer code solves two couple poisson equations in general curvilinear coordinate. For solving elliptic differential equations we have been used TTM (Thompson, Thames & Mastin) algorithm. The user of this code can also solve laplace's equation instead of poisson's equation. By using this new method, we can find electric field distribution around any complex electric equipment.

### 1. Introduction

In this paper, we can obtain electric field intensity and also equipotential lines around any arbitrary shape electrodes with TTM (Thompson, Thames & Mastin) algorithm. With this computer code we can solve two couple poisson or laplace equations in general curvilinear coordinate. For finding electric field intensity and also equipotential lines around any electrode, we transfer actual field or actual coordinate system (surface of any arbitrary shape electrode) and also its outer boundary to rectangular calculating field or rectangular coordinate system. Solving of poisson and also laplace equations are easy in rectangular field than actual field [1].

As we know that, by using conformal mapping, we can solve electric fields and also equipotential lines around some definite electrodes [2]. In conformal mapping method we define any complex transfer function and then, by using this function, for every point in actual complex  $z$ -plane with coordinates  $x$  and  $y$  we can find one or many points in an other complex  $w$ -plane with coordinates  $u$  and  $v$ . The relationship between points in  $z$ -plane and  $w$ -plane have been determined by complex analytic function  $w = f(z)$ . This complex analytic function is named transfer function also. Since definition of transfer function is not possible for all type electrode system therefore in general we can use conformal mapping technique under some special conditions [2].

In this computer code, similar to conformal mapping technique for calculating electric field and equipotential lines around any arbitrary shape electrodes first of all we transfer actual field to

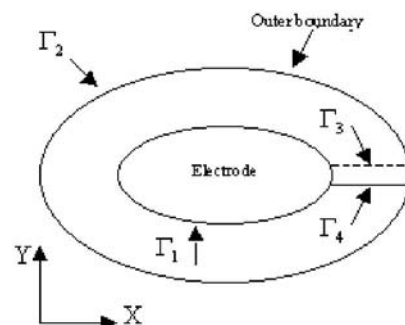
rectangular calculating field and after solving laplace or poisson equations in rectangular coordinate system we again transfer the results to actual field also. The constraints of the conformal mapping method have been not seen in this code. In mathematical method of this computer program, we will explain transfer function briefly.

In the past several methods such as finite difference, finite element, boundary element and charge simulation have been used for solving electric field distribution [1-9]. According to some difficulties in the above mentioned different methods we have obtained a paper that is comparing finite difference, finite element and charge simulation methods with each other [4]. In finite difference method if the shape of the electrode is complicated therefore the results obtained by this method will have a large errors. In this new method we can analysis electric field and also equipotential lines around any arbitrary shape electrode with minimum errors.

### 2. Mathematical Discussion

Since calculation of electric field intensity in rectangular plane is quite simple therefore In this computer program we transfer actual computational field and its outer boundary to any computational rectangular plane.

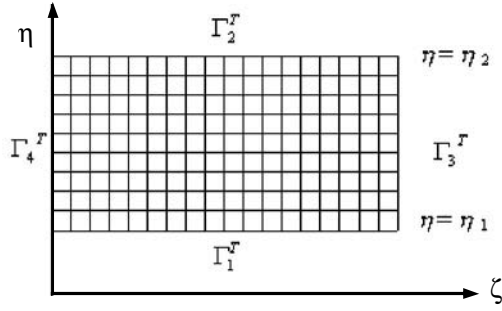
Henceforth, we show actual field with  $D$  and calculating field with  $D^T$ . Figure 1 shows actual field (body or electrode and its outer boundary) in complex  $z$ -plane with  $x$  and  $y$  coordinates.



**Figure 1:** One electrode and its outer boundary (actual field)

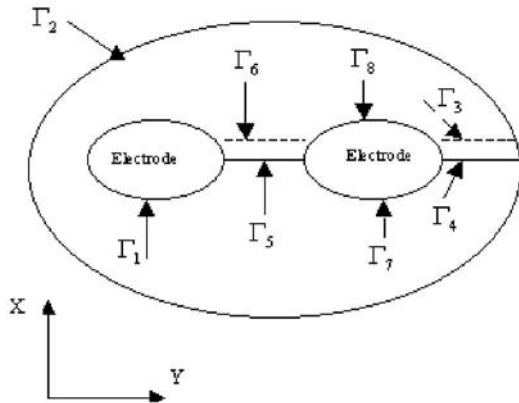
In figure 1,  $\Gamma_1$  shows body or electrode surface and  $\Gamma_2$  shows outer boundary in the computational field, in this figure also we assume two cutting lines or curves

$\Gamma_3$  and  $\Gamma_4$  respectively these hypothesis lines or curves connect body or electrode to outer boundary in actual field. Figure 2 shows calculating field in complex w-plane with coordinates  $\zeta$  and  $\eta$  respectively. In general, electrode surface and outer boundary transfer to  $\eta$  constant lines ( $\eta_1$  and  $\eta_2$ ) that is shown in figure 2. In rectangular computational field, hypothesis lines or curves in Fig. 1 have been shown by lines  $\Gamma_3^T$  and  $\Gamma_4^T$ . Notice that  $\Gamma_3$  and  $\Gamma_4$  in actual field have been coincide to each other therefore coordinates  $X(\zeta, \eta)$  and  $Y(\zeta, \eta)$  for  $\Gamma_3$  and  $\Gamma_4$  will be equal.



**Figure 2:** Computational field of one electrode and its outer boundary

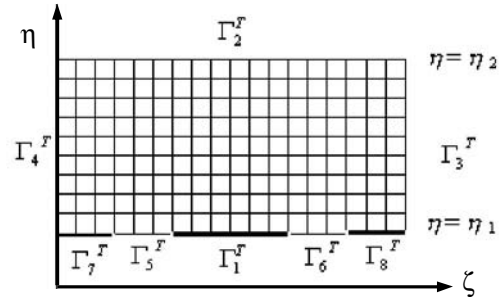
In the above discussion we assumed only one electrode or body in actual field. In this code we can assume many electrodes in actual field therefore by this new method electric field intensity and equipotential lines around three phase transmission line can be investigated. Figure 3 shows actual field and figure 4 shows computational field of two electrodes configuration.



**Figure 3:** Two electrodes and their boundary (actual field)

From the point of view of mathematics, transfer from actual field to computational field and vice versa can be shown by the following equations.

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} \zeta(X, Y) \\ \eta(X, Y) \end{bmatrix} \quad (1)$$



**Figure 4:** Computational field of two electrodes and their outer boundary

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X(\zeta, \eta) \\ Y(\zeta, \eta) \end{bmatrix} \quad (2)$$

The matrices of this transformation are as follows:

$$J_1 = \begin{bmatrix} \zeta_X & \zeta_Y \\ \eta_X & \eta_Y \end{bmatrix}, \quad J_2 = \begin{bmatrix} X_\zeta & X_\eta \\ Y_\zeta & Y_\eta \end{bmatrix} \quad (3)$$

According to above equation we can say that  $[J_1] = [J_2]^{-1}$ . In general the elements of matrix  $J_1$  are obtained as follows:

$$\zeta_X = Y_\eta / J, \quad \zeta_Y = -X_\eta / J, \quad (4a)$$

$$\eta_X = -Y_\zeta / J, \quad \eta_Y = X_\zeta / J \quad (4b)$$

In the above relationships  $J$ , shows the determinant of matrix  $J_2$ , following equation shows the determinant of matrix  $J_2$ :

$$J = \det[J_2] = X_\zeta Y_\eta - X_\eta Y_\zeta \quad (5)$$

The solved equations in this computer program are laplace or poisson. At first step we consider laplace equations solution and at the second step we investigate poissons equations solution. In general, in curvilinear coordinate system laplace equations are shown as follows:

$$\zeta_{XX} + \zeta_{YY} = 0 \quad (6)$$

$$\eta_{XX} + \eta_{YY} = 0 \quad (7)$$

with respect to Dirichlet boundary conditions can be obtained following equations:

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} \zeta_1(X, Y) \\ \eta_1 \end{bmatrix}, \quad [X, Y] \in \Gamma_1^T \quad (8)$$

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} \zeta_2(X, Y) \\ \eta_2 \end{bmatrix}, \quad [X, Y] \in \Gamma_2^T \quad (9)$$

In equations (8) and (9),  $\zeta_1$  and  $\zeta_2$  are known functions that are obtained from curves  $\Gamma_1$  and  $\Gamma_2$  (electrode surface and outer boundary) separately. In computational field (rectangular coordinate), we use following equations:

$$\alpha X_{\zeta\zeta} - 2\beta X_{\zeta\eta} + \gamma X_{\eta\eta} = 0 \quad (10)$$

$$\alpha Y_{\zeta\zeta} - 2\beta Y_{\zeta\eta} + \gamma Y_{\eta\eta} = 0 \quad (11)$$

In equations (10) and (11), constants  $\alpha$ ,  $\beta$  and  $\gamma$  can be obtained from following equations:

$$\alpha = X_{\eta}^2 + Y_{\eta}^2 \quad (12)$$

$$\beta = X_{\zeta}X_{\eta} + Y_{\zeta}Y_{\eta} \quad (13)$$

$$\gamma = X_{\zeta}^2 + Y_{\zeta}^2 \quad (14)$$

Under transferred boundary conditions, we obtain following equations:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} f_1(\zeta, \eta_1) \\ f_2(\zeta, \eta_1) \end{bmatrix}, \quad [\zeta, \eta] \in \Gamma_1^T \quad (15)$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} g_1(\zeta, \eta_2) \\ g_2(\zeta, \eta_2) \end{bmatrix}, \quad [\zeta, \eta] \in \Gamma_2^T \quad (16)$$

Functions  $f_1$ ,  $f_2$ ,  $g_1$ ,  $g_2$  in equations (15) and (16) are well defined, since these functions are the coordinates of region ( $\Gamma_1$  and  $\Gamma_2$ ) that we define in the actual field.

Notice that the systems of the above equations are quasi-linear and elliptic, therefore the solution of these functions are very complicated than laplace equations in actual field coordinates, but in this code we solve these equations in rectangular coordinate system that make easy our calculations.

If we have two electrodes in actual field for second electrode also we can obtain following equations:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} h_1(\zeta, \eta_1) \\ h_2(\zeta, \eta_1) \end{bmatrix}, \quad [\zeta, \eta_1] \in \Gamma_7^T \quad (17)$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} o_1(\zeta, \eta_1) \\ o_2(\zeta, \eta_1) \end{bmatrix}, \quad [\zeta, \eta_1] \in \Gamma_8^T \quad (18)$$

In general, in curvilinear coordinate system poisson equations are shown as follows:

$$\zeta_{XX} + \zeta_{YY} = \varepsilon_1 \quad (19)$$

$$\eta_{XX} + \eta_{YY} = \varepsilon_2 \quad (20)$$

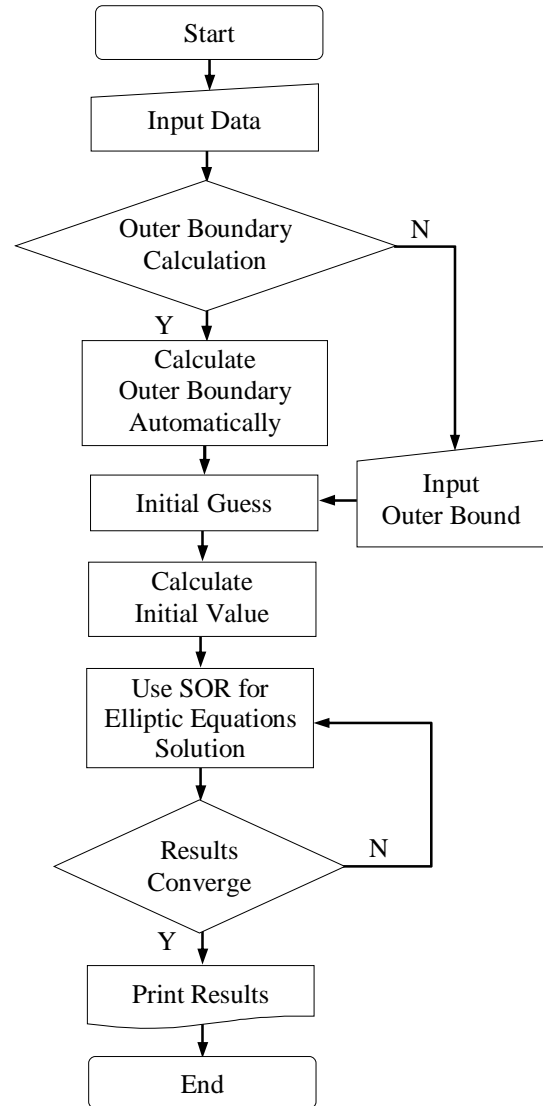
In equations (19) and (20),  $\varepsilon_1$  and  $\varepsilon_2$  can be equal or different. According to above discussion, we obtain following equations in transformed field or plane.

$$\alpha X_{\zeta\zeta} - 2\beta X_{\zeta\eta} + \gamma X_{\eta\eta} + J^2(\varepsilon_1 X_{\zeta} + \varepsilon_2 X_{\eta}) = 0 \quad (21)$$

$$\alpha Y_{\zeta\zeta} - 2\beta Y_{\zeta\eta} + \gamma Y_{\eta\eta} + J^2(\varepsilon_1 Y_{\zeta} + \varepsilon_2 Y_{\eta}) = 0 \quad (22)$$

### 3. Flowchart of The Code

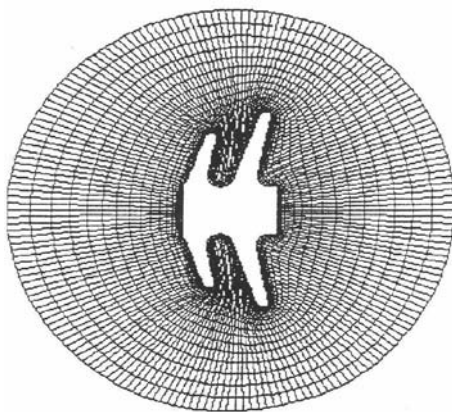
The Following figure shows the flowchart of this computer code. As we have mentioned previously, by this computer code we can analysis electric field intensity and also equipotential lines around any arbitrary shape electrode.



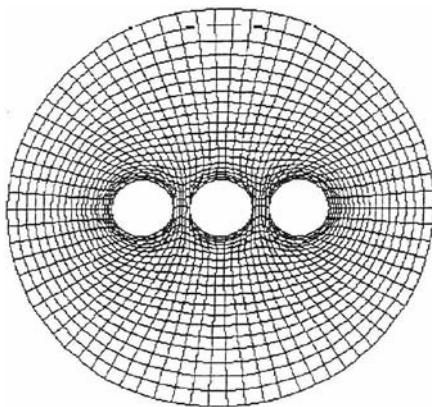
**Figure 5:** Flowchart

#### 4. Comparison of The Results

Following figures shows the electric field intensity and also equipotential lines around any arbitrary shape electrode. In Figure 6 electric field intensities and also equipotential lines around a complicated shape electrode, and also in Figure 7 electric field and equipotential lines around three conductors with the same shape have been obtained by this new method. Comparison these results with analogue electrode systems electric field intensity, that obtained with different methods in given references, shows that we can use this new algorithm for analyzing electric field intensity around any arbitrary shape electrode or conductors with the same or different configuration.



**Figure 6:** Electric field intensities and equipotential lines around arbitrary shape electrode



**Figure 7:** Electric field intensities and equipotential lines around three conductors

#### 5. Conclusion

We have described here a new computer program with the ability to solve poisson's and laplace's equations. With this new method we can calculate electric field intensity and also equipotential lines around any arbitrary shape electrode or conductors with the same or different configurations. As we know that in high voltage engineering, analysis of electric field around

any equipment has very important role in the design of that equipment therefore, we can use this new computer code in the design and construction of any HV equipment that are used in electric industry. Concerning to precision and ability of this new method we can use this code also in research and educational activities.

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